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Contrast and entailment: Abstract logical relations constrain how 2- and 3-year-old children interpret unknown numbers

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ABSTRACT

Do children understand how different numbers are related before they associate them with specific cardinalities? We explored how children rely on two abstract relations – *contrast* and *entailment* – to reason about the meanings of 'unknown' number words. Previous studies argue that, because children give variable amounts when asked to give an unknown number, all unknown numbers begin with an existential meaning akin to *some.* In Experiment 1, we tested an alternative hypothesis, that because numbers belong to a scale of contrasting alternatives, children assign them a meaning distinct from *some*. In the "Don't Give-a-Number task", children were shown three kinds of fruit (apples, bananas, strawberries), and asked to *not* give either *some* or a number of one kind (e.g. *Give everything, but not [some/five] bananas*). While children tended to give zero bananas when asked to *not give some*, they gave positive amounts when asked to not give numbers. This suggests that contrast – plus knowledge of a number's membership in a count list – enables children to differentiate the meanings of unknown number words from the meaning of *some*. Experiment 2 tested whether children's interpretation of unknown numbers is further constrained by understanding numerical *entailment* relations – that if someone, e.g. *has three*, they thereby also have two, but if they *do not have three*, they also do not have four. On critical trials, children saw two characters with different quantities of fish, two apart (e.g. 2 vs. 4), and were asked about the number in-between – who either *has* or *doesn't have*, e.g. *three.* Children picked the larger quantity for the affirmative, and the smaller for the negative prompts even when all the numbers were unknown, suggesting that they understood that, whatever *three* means, a larger quantity is more likely to contain that many, and a smaller quantity is more likely not to. We conclude by discussing how contrast and entailment could help children scaffold the exact meanings of unknown number words.

1. Introduction

As adults, we are able to think and talk about abstract concepts like *integer*, *atom*, and *density*. We use these words to describe and explain our perceptual experience of the world. To learn the meanings of such words, however, it is not enough to observe the world through our senses and associate percepts with labels – our eyes and ears are simply not equipped to detect integers, atoms, or density. Moreover, no simple concatenation of perceptual building blocks can supply the content of our most abstract mental representations. Even in domains for which humans have relatively robust perceptual systems – like time, space, and number – these representations are profoundly limited and noisy, a fact which likely motivated earlier generations of humans to create external symbolic systems for describing and explaining the world (Barner, in press). According to some accounts, this process of constructing abstract symbolic systems, found in human history, is recapitulated in child development, and involves a form of "bootstrapping", wherein children begin by building structures to describe phenomena that are available to perception, and then leverage these new structures to construct representations that are ever more complex and divorced from perception (Carey, 2009; MacNamara, 1972; Braine, 1992).

One prominent suggestion for such a mechanism is what Carey (2009) calls Quinian bootstrapping, based on a metaphor first described by Quine (1960):

> "The child scrambles up an intellectual chimney, supporting himself against each side by pressure against the others. Conceptualization on any considerable scale is inseparable from language, and our ordinary language of physical things is about as basic as language gets."
> (Quine, 1960, p. 93)

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On Carey's (2009) account, the walls of the Quinian chimney take the form of so-called placeholder structures – words that children hear used by adults in their input, but for which they don't yet have meanings. Learning these structures – e.g., how duration words like *second* and *minute* are related, or how the number words are ordered – restricts children's inferences about the meanings of the individual words contained within them (Tare, Shatz, & Gilbertson, 2008). For example, to learn the meaning of a word like *hour*, children first learn that *hour* belongs to a class of alternatives including *second, minute,* and *day*, then learn how these words are ordered with respect to duration, and then learn the precise relations that define them – e.g., that an hour is 60 min, which in turn can be defined as 60 s (Shatz, Tare, Nguyen, & Young, 2010; Tillman & Barner, 2015). On this view, word learning – and the process of conceptual development that accompanies it – is an iterative process wherein learning words and their relations to one another allows children to entertain increasingly sophisticated meaning hypotheses, which are defined in terms of relations between symbols and concepts themselves, rather than in terms of mappings to simple perceptual phenomena (see Carey, 2009; Wagner, Tillman, & Barner, 2016; and Block, 1987 for a more general defense of inferential role semantic theories).[1]

1.1. Natural number

In this paper, we investigate children's understanding of the logical relations between words in the count list, and whether learning such relations might constrain the acquisition of individual number word meanings. Although various case studies have been used to explore the Quinian bootstrapping hypothesis (Carey, 2009), number word learning provides an especially compelling area of inquiry for at least two reasons. First, although humans have perceptual representations of objects and number from the earliest moments of life (Feigenson, Dehaene, & Spelke, 2004), these representations do not provide the content needed to learn large exact natural numbers, like 10 or 452. Representations of objects and approximate magnitudes lack the logical content that number words must ultimately encode, such as exact equality, and the successor function (i.e., that for every natural number, n, there is a successor defined as $n + 1$). These facts suggest that perception of magnitudes alone cannot provide the meanings of number words (for review, see Barner, 2017, in press; Laurence & Margolis, 2005), and that the counting structure itself plays a role in their construction.

A second reason that number words provide a compelling test of Quinian bootstrapping is that by around the age of 2, children in the US learn a partial count list – up to 5 or 10 – before they have learned the meanings of any of these words, much like they learn to recite letters of the alphabet (Fuson, 1988; Gelman & Gallistel, 1978). Thus, children construct a placeholder structure as a first step in the learning process. At roughly six month intervals thereafter, children learn the meanings of *one, two,* and *three*, one at a time, and always in that sequence (see Le Corre, Brannon, Van de Walle, and Carey, 2006; Wynn, 1990, 1992). Children at these different stages are collectively known as "subset-knowers", because they know the meanings of only a subset of the numbers in their count list. Thus, up until this point, children's knowledge of counting structures outstrips their knowledge of what these structures mean, and what cardinalities the individual number words denote – a precondition for the use of these structures in a bootstrapping process. At around the age of 3½ to 4 years, US children appear to realize that the counting routine can be used to label and generate sets (Carey, 2004, 2009; Sarnecka & Carey, 2008; Schaeffer, Eggleston, & Scott, 1974). At this stage, children are known as Cardinal Principle knowers (or CP-knowers), because they can use counting to construct sets for all numbers in their count list (e.g., counting and giving eight objects upon request). Finally, around two years later, US children appear to discover *why* counting works in this way and, in particular, how numbers are related via the successor principle. When told the cardinality of a set – e.g., *I have five frogs* – children around 5½ years old are able to predict how many frogs will result if one more is added to the set, and know that this function can be applied indefinitely, resulting in a potential infinity of natural numbers (Cheung, Rubenson, & Barner, 2017; Davidson, Eng, & Barner, 2012; Evans, 1983; Gelman, 1980; Hartnett, 1991; Hartnett & Gelman, 1998; Spaepen, Gunderson, Gibson, Goldin-Meadow, & Levine, 2018). This series of stage-like transitions, also found in various other cultures (Almoammer et al., 2013; Barner, Libenson, Cheung, and Takasaki, 2009; Le Corre, Li, Huang, Jia, & Carey, 2016; Piantadosi, Jara-Ettinger, & Gibson, 2014; Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007). This suggests that children's learning of larger number words begins with a placeholder structure that is gradually filled in with different components of meaning, allowing children to ascend a conceptual chimney as each new brick is laid.

Given that children do not understand counting early in development and take years to understand its precise logic, how could the structure of counting play a role in learning individual number word meanings? One clue comes from a study by Wynn (1992), who asked whether, as in other cases of word learning, children expect number words to respect Clark's "Principle of Contrast" – i.e., that a new word is likely to have a different meaning than words they already know (Clark, 1987; Carey & Bartlett, 1978; Golinkoff, Mervis, & Hirsh-Pasek, 1994; Markman, 1990; Woodward & Markman, 1998). In her study, Wynn (1992) showed that when 1-knowers are presented with a choice between two sets – e.g., 1 balloon vs. 5 balloons – they select the larger set when asked to point to *five balloons*, despite not yet knowing the meaning of *five*. This suggests that children used their prior knowledge of *one* to restrict their interpretation of *five* by assuming that the two numbers could not denote the same quantity. Critically, however, children did not assume that known number words (e.g., *one* in this example) contrast with just *any* other word: When shown the same comparison of 1 vs. 5 and asked to point to *blick balloons*, children pointed randomly, suggesting that they contrasted *one* and *five,* but not *one* and *blick* (Wynn, 1992; see also Barner & Bachrach, 2010; Clark, 1987, 1988).

While Wynn's study shows that children assume that number word meanings contrast, it leaves open what they think these contrasting meanings might be. On one account, proposed by Sarnecka and Gelman (2004), subset knowers assume that members of the count list denote precise cardinalities, each of which contrasts with the others, but simply don't know which numbers denote which cardinalities. In support of this claim, Sarnecka and Gelman presented subset knowers with a set transformation study, in which a quantity beyond their knower level (e.g., five frogs) was placed in a container and labeled (e.g., *Here are five frogs*). The hidden set was then modified by adding an additional object, and children were asked whether there were now five (the original number) or six (a novel number). Sarnecka and Gelman found that when the set was changed children preferred the novel number, consistent with the belief that changes in cardinality require changes in number words. Other studies, however, suggest that children's performance on this task may not reflect knowledge of number, but instead an assumption that contrast applies whenever some significant transformation of a referent takes place (for discussion, see Brooks, Audet, & Barner, 2013; Condry & Spelke, 2008; Izard, Streri, & Spelke, 2014; Sarnecka & Wright, 2013). For example, Brooks et al. (2013) found an identical pattern of results when children were shown similar physical

[1] A closely related process has been proposed within Bayesian learning frameworks. In these frameworks, the iterative interplay between specific hypotheses about a given concept (e.g. the meaning of *day*) and more abstract overhypotheses about the relation between concepts (e.g. the relation between measurement units of time) constrains both kinds of hypotheses, and enables the learning of more complex conceptual structures than would otherwise be possible (see Dewar & Xu, 2010; Goodman, Ullman, & Tenenbaum, 2011; Kemp, Perfors, & Tenenbaum, 2007; Tenenbaum, Griffiths, Kemp, and Goodman, 2011).

manipulations, but using novel nouns like *blicket*, rather than number words. This result, and others that avoid presenting children with contrasting labels to describe transformed sets, suggest that Sarnecka and Gelman's task may reflect a general sensitivity to contrast in labeling, rather than to the semantics of number words, while leaving open the possibility that other tasks may detect more specific semantic knowledge (Condry & Spelke, 2008; see also the two set tasks of Sarnecka & Gelman, 2004, and Sarnecka & Wright, 2013; see also McGarrigle & Donaldson, 1974, for a similar pragmatic explanation of Piagetian conservation failures).

In contrast to Sarnecka and Gelman (2004), others have argued that children initially assign number words more general, set relational meanings, like other quantity expressions in natural language (Barner, Chow, & Yang, 2009; Bloom & Wynn, 1997; Carey, 2004, 2009; Clark, 1970; Clark & Nikitina, 2009). For example, in his discussion of how children learn quantifiers like *more* and *less,* Clark (1970) argued that children initially assume that all quantity expressions have a type of existential meaning, like the word *some*. According to Clark,

> In the first stage, *more* and *less* both mean "a quantity of" or "some." A question people might commonly ask a child is "Do you want more food?"… He would encounter *less* also as "some" since it occurs as a single "adjective" modifying *food*. The senses of "quantity of" and "some," of course, are equivalent to the nominal use of *much* (Clark, 1970, p. 272)

Similarly, Carey (2004) and Clark and Nikitina (2009) have both argued that children interpret number words beyond their knowledge like the plural existential *some*, e.g., using the word *two* whenever more than one object is present (though see Barner, Lui, & Zapf, 2012). On this type of account, children make minimal assumptions regarding the meanings of quantity expressions, and do not initially use their knowledge of the count list to differentiate them from other quantity expressions in natural language.

A problem with adjudicating between these different theories of number word learning is that, like the controversial set transformation task of Sarnecka and Gelman, most existing methods fail, in a principled way, to generate diagnostic patterns of data. For example, all theories make the same prediction regarding Wynn's Give-a-Number task: Whether children believe that *five* denotes a cardinality or they believe that it has an existential meaning like *some*, a 1-knower should give any random amount greater than one when asked to give *five*. Similarly, all theories make the same prediction regarding Wynn's "Point to N" task. When a 1-knower is shown a comparison of 1 object vs. 5 objects and asked to point to the set with *five*, they should point to the larger set on the basis of contrast alone, whether they believe that *five* denotes a specific cardinality or has an existential meaning. Other tasks used in the literature, e.g., that ask children to label or count sets, encounter similar limitations. In general, any specific cardinality like five, seven, or seventeen can be described by either a number word or an existential quantifier like *some*.

In the present study, we sought to circumvent this problem by creating two new tasks. First, Experiment 1 addressed the limits of the Give-a-Number task by creating what we call the "Don't Give-a-Number" Task. Our creation of this task was driven by the following observation: While *five* and *some* can refer to the same set of five objects, these words behave differently in sentences involving negation. For example, consider the sentences in (1) and (2):

(1) Give Cookie Monster something to eat, but don't give him some of the cookies.
(2) Give Cookie Monster something to eat, but don't give him five of the cookies.

Whereas the sentence in (1) favors a reading in which Cookie Monster should receive no cookies, the sentence in (2) favors giving some amount of cookies, but not exactly five. For adults, other readings are also possible, but as we report, these do not generally arise for children (see also Musolino & Lidz, 2006; Viau, Lidz, & Musolino, 2010).[2] At least for adults, although *some* and *five* might refer to the same quantity of objects, their respective negations do not. However, if young children assign the unknown number *five* the same meaning as the existential *some*, they should interpret *not five* just the same as *not some* – as meaning *none*. If, on the other hand, children treat the count list as a set of alternatives and use it to constrain their interpretation of unknown numbers – consistent with Quinian bootstrapping – then they may reason that an amount described as "*not five*" is compatible with other, alternative, number words like *six, seven, eight,* etc.

1.2. Number and entailment

Children might use the structure of the count list to infer that each number picks out a different quantity, consistent with Quinian bootstrapping. However, it is also possible that the count list plays a still stronger role in learning. Unlike many other classes of words, like those that label colors or animals, the count list is an ordered scale of alternatives: Numbers later in the count list denote greater quantities, and quantities are related to each other by set inclusion, such that larger sets contain all smaller ones. Adults are clearly sensitive to this fact. For example, if I know that I have three quarters in my pocket and I need two for the parking meter, I do not need to count again to see whether I have two. I know that if I have three I must also have two, because having three *entails* having two. However, there is no entailment relation in the opposite direction, from small numbers to greater amounts: Having three does not entail having four. Furthermore, negation reverses this entailment pattern. If I need three quarters for the meter and do *not* have three, I thereby also know that I do not have four, or five, although I might have one or two. These kinds of "asymmetric" entailment relations hold in many domains – from quantification (if *every X is Y,* then *some X is Y*) to adjectival modification (if something is *very nice,* it is also *nice*), to natural kinds (if something is a *dog*, it is also an *animal*), among many others. Entailment relations are found in all languages and are a hallmark of the structure of the human conceptual system (see Horn, 1972).

In the following two experiments, we explore children's understanding of the abstract logical relations that hold between numbers – an understanding that might serve as input into a process of Quinian bootstrapping. In Experiment 1, we test whether children interpret unknown number words differently than other quantifiers. Specifically, we test whether children's knowledge that number words contrast with one another leads them to interpret numbers distinctly from other quantifiers, and to associate them with different physical quantities: Whereas *not giving five* leaves open many alternative numbers, *not giving some* does not. In Experiment 2, we test whether children understand the entailment relations expressed by number words. Do they know that if a set has a given number, *N*, then there may actually be more than N items in it, but not less than N? And by extension, do they know that *not* having *N* entails that there are fewer than N items? Importantly, this knowledge does not depend on knowing the meaning of any individual number. Therefore, Experiment 2 further asks whether children might have such knowledge even when they do not yet know the meaning of *N,* and thus whether knowledge of entailment relations might precede knowledge of cardinal meanings in acquisition.

[2] Adults might compute a scalar implicature, interpreting *some* as *some-but-not-all*, in which case giving all objects is a legitimate response. They might also interpret negation with inverse scope, such that there is a set of objects that should not be given. In contrast, young children rarely if ever compute spontaneous scalar implicatures over quantifiers (Noveck, 2001; Papafragou & Musolino, 2003) or inverse scope (Musolino & Lidz, 2006; Viau et al., 2010). Consistent with this, children's behavior in Experiment 1 suggests they neither computed implicatures nor inverse scope (they did not tend to *leave* the requested amount, see Fig. A2).

2. Experiment 1

We tested children's interpretation of both known and unknown numbers using the "Don't Give-a-Number" task. Although it is unclear when English-speaking children begin to understand *don't,* recent work shows that they comprehend the words *no, not,* and *didn't* as markers of logical negation shortly after they turn two (Austin, Theakston, Lieven, & Tomasello, 2014; Feiman, Mody, Sanborn, & Carey, 2017; Reuter, Feiman, & Snedeker, 2018). Children at this age are typically non-knowers or 1-knowers, with many numbers still left to learn. We therefore took age two as a starting point and aimed for a sample including all knower levels for children up to age four, when most children are CP-knowers, in order to test how children will interpret unknown number words in combination with negation.

Children were presented with three sets of objects – strawberries, bananas, and oranges – to feed a hungry puppet. They were then told to give the puppet everything, "*but don't give him X*"*,* where X was some amount (*five bananas*, *some of the bananas*, etc.). This task allowed us to determine whether children initially interpret number words like the existential quantifier *some* (or the plural marker *–s*), or if they instead use their status as contrasting alternatives to ascribe them different meanings, and associate them with different physical quantities (Sarnecka et al., 2007). These two possibilities clearly dissociate under negation: When asked not to give *three*, a child who believes *three* is an existential should give nothing (*not some* = *none*), whereas a child who believes that all numbers contrast in quantity should interpret the request relative to other alternative requests that might have been made, but were not (since not giving *three*, whatever that means, leaves open the possibility of giving *one*, *two*, *four*, or *five*, etc.).

2.1. Method

2.1.1. Participants

We tested 90 English-speaking children with the goal of testing at least 15 children per knower-level (an additional 32 failed to complete at least one trial for every word).[3] Because a child's knower level group could not be known before testing them, and we did not discard data, this resulted in more children in some groups than in others. We excluded children who gave the same response on more than half of the trials. Seventeen children were excluded on this criterion (8 non-knowers, M age: 2;9, 3 1-knowers, M age: 3;4, two 2-knowers, M age: 3;0, two 3-knowers, M age: 3;1, one 4-knower, age: 3;0, and one CP-knower, age 3;8).[4] The remaining 73 children were aged 2;4 to 4;1 (*M* = 3;4). Of these, 13 were identified by Wynn (1990) "Give-a-Number" task as non-knowers (*M* = 2;10, 2;4–3;3; 8 male). There were 39 subset-knowers, including 10 1-knowers (M = 3;0, 2;6–3;8, 4 boys), 16 2-knowers (M = 3;2, 2;11–3;8, 5 boys), 11 3-knowers (M = 3;6, 3;1–3;9, 3 boys), 2 4-knowers (both 3;10, both girls). Finally, there were 21 CP-knowers (*M* = 3;6, 2;11–4;1; 4 male). Children were either recruited by phone and brought into the lab or were recruited and tested at daycares and museums in the San Diego area. All children received a token gift for participating.

2.1.2. Materials and procedure

The experimenter first administered Wynn (1990) "Give-a-Number" task to determine the child's knower-level. Then, they introduced the child to a puppet named Farmer Brown. The experimenter began by placing a paper plate, five plastic bananas, five plastic strawberries, and five plastic oranges in front of the child. Children who could not identify the names of the fruits were familiarized with their labels. To ensure that the child could identify each fruit and complete the task, the experimenter requested that the child put each type of fruit onto the plate, one at a time. If the child failed to do so, the experimenter provided feedback and asked the child to try again. If the child failed once more, the task was discontinued.

On each trial, children were told that the puppet was hungry and were then given a prompt. To ensure that behavior did not depend on the exact wording of the prompt, children were randomly assigned to one of two prompts, told either, *Give Farmer Brown everything, but don't give him [1 banana],* or, *Farmer Brown likes bananas and strawberries and oranges, but don't give him [1 banana]*. There were 12 trials in total, two for each number (*one*, *two*, *five*) and quantifier (*a*, *some*, *all*). The trials were presented in two quasi-random orders, counterbalanced across participants. No quantifier or number was ever presented twice in a row, and the types of fruit targeted were evenly distributed across trials.

2.2. Results and discussion

Raw data and statistical code are available at https://osf.io/ht6wd/. Because we had no predictions that were specific to particular knower levels, but only regarding unknown vs. known numbers, the analyses reported below group all subset-knowers together. CP-knowers were separated from subset-knowers, since no numbers were unknown to them. For the interested reader, we report breakdowns by knower-level within the subset-knower group in footnotes, with figures in Appendix A.

In order to explore whether children assign number words an existential reading, we compared their proportion of "zero" responses across request types. If children differentiate *some* and *a* from number words, they should give zero more often when asked not to give either *some* or *a* than when asked not to give a number. We further explore whether children differ along this dimension depending on whether they have assigned an exact meaning to all numbers (CP-knowers), some subset of numbers (subset-knowers), or have not yet learned any exact number meanings (non-knowers). We built a mixed effects logistic regression (R version 3.5.1 (2018–07–02); lme4 1.1.18.1), with Giving Zero as a binary dependent variable, and two categorical fixed effects: Number Knowledge (non-knowers, subset-knowers, and CP-knowers), and Existential, (whether the prompt was existential - *some* or *a* – or a number – *one*, *two*, or *five*) along with a random intercept and slope of the Existential variable by subjects, and a random intercept by items.[5] This analysis found a highly significant effect of Existential ($\chi^2 = 53.78$, $p < 0.001$), and a significant interaction of Number Knowledge by Existential ($\chi^2 = 8.32$, $p = 0.016$).[6,7] Unpacking this

[3] Of these, 5 children did not finish the Give-N task and could not be classified by knower-level. Of the remainder, there were 14 non-knowers (M age: 2;10), four 1-knowers (M age: 3;0), four 2-knowers (M age: 3;1), one 3-knower (age: 3;7), and four CP-knowers (M age: 3;6).

[4] Ten children gave all of each type of fruit on more than half the trials. Seven gave exactly one of each type of fruit on more than half the trials.

[5] The random effects specification followed the guidelines of Matuschek et al. (2017). P-values for all models were derived by a Wald Type II Chi-square test, comparing a model including the factor to a minimally different model excluding it.

[6] Children received all questions in one of two prompt frames: either, *Give Farmer Brown everything, but don't give him [1 banana]*, or *Farmer Brown likes bananas and strawberries and oranges, but don't give him [1 banana])*. The effect of the Existential variable was significant following either prompt (1st: $\chi^2 = 32.92$, $p < 0.001$; 2nd: $\chi^2 = 61.61$, $p < 0.001$). A separate regression with Existential, Prompt, and their interaction as fixed effects found main effects both of Existential ($\chi^2 = 55.64$, $p < 0.001$), and of Prompt ($\chi^2 = 12.27$, $p < 0.001$), and a marginal interaction ($\chi^2 = 2.95$, $p = 0.086$). There was also a difference in the number of excluded children between prompts (15 kids excluded for the first, 29 for the second; exact Chi-square test: $\chi^2 = 6.52$, $p = 0.011$). Thus, while the first prompt was generally easier for children to understand across the different quantifier requests, children differentiated existentials from non-existentials given either prompt.

[7] Overall, there was a high rate of exclusion in this task (38% summing over both exclusion criteria), reflecting its difficulty. However, the omnibus significant effect of the Existential variable on the rate of zero-giving remains even

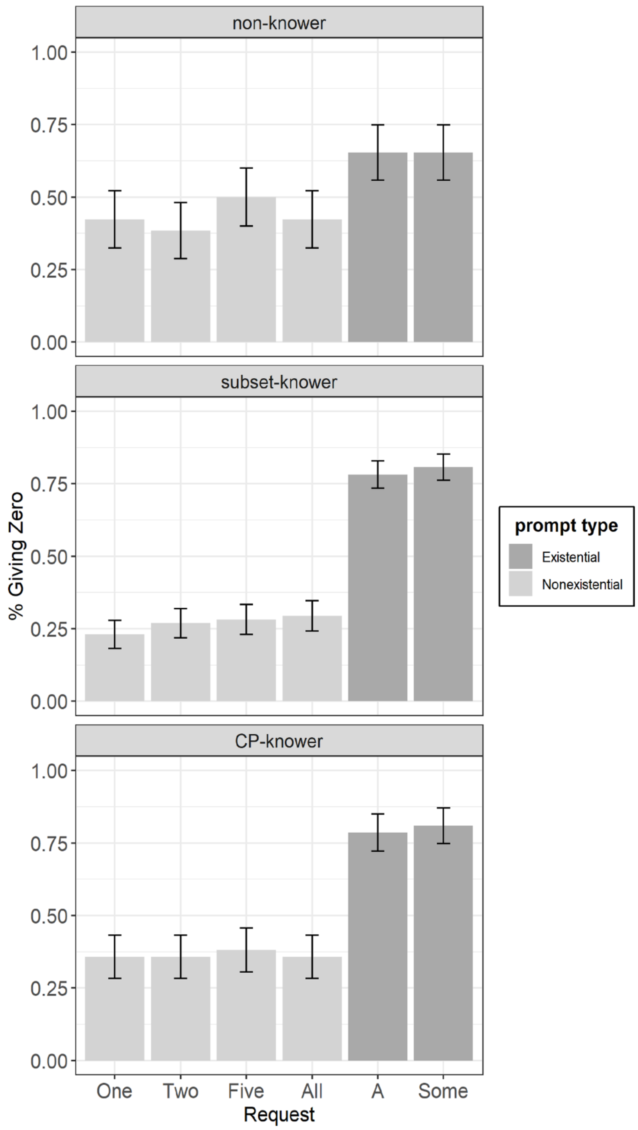


Fig. 1. Proportion of trials on which participants gave 0 of the target items in response to each type of request. Error bars show ± 1 standard error, averaged by participant.

interaction by levels of Number Knowledge to look at the effect of Existential within the non-, subset-, and CP-knowers separately, each group was individually more likely to give zero for the existential forms *a* and *some* than for the numbers (Fig. 1; non-knowers: $\chi^2 = 6.28$, $p = 0.012$; subset-knowers: $\chi^2 = 40.58$, $p < 0.001$; CP-knowers: $\chi^2 = 14.86$, $p < 0.001$.[8] Thus, all groups of children clearly differentiated number words from existential quantifiers, in keeping with the hypothesis that membership in a scale of contrasting alternatives – i.e., the count list – affects their interpretation.

We next probed whether this finding was true of unknown numbers in particular. While we found that even non-knowers, who know no number words, differentiate numbers from *some* and *a*, we sought a stronger test by specifically asking whether subset knowers treated known and unknown numbers differently. We modeled the effect of Prompt Knowledge (whether the mentioned number N was above the child's knower level or instead at-or-below it) within each group of subset knowers separately (1–2-, and 3-knowers). Within each of these three groups, there was no significant difference in the rate of zero-giving between known and unknown numbers (all *ps* > 0.19), although there was a difference between unknown numbers and the existentials *a* and *some* (see Fig. 1) suggesting that children treated unknown numbers the same as known numbers and differently from existentials, giving zero less often when asked not to give a number than when asked not to give *some*, whether the number was known or unknown.

Also, as Fig. 1 shows, the quantifier *all* patterned with the numbers: Children rarely gave zero objects when asked not to give *all*. *All* served as a control to check that children did not simply give zero objects to any non-numeric quantifier prompt, but gave none selectively, only when asked for *not a* or *not some*. In a model including Number Knowledge (non-knowers, subset-knowers, CP-knowers) and Quantifier (*all* vs. the existentials *some* and *a* grouped together), children gave zero significantly more often when asked not to give an existential, than when asked not to give *all* ($\chi^2 = 8.37$, $p = 0.004$), with no main effect of Number Knowledge ($\chi^2 = 1.55$, $p = 0.462$), but a significant interaction ($\chi^2 = 6.04$, $p = 0.049$), reflecting non-knowers' weaker differentiation between prompts. Critically, children's differentiating existentials from *all* is not due to their failing to comprehend *all*: Although *all* resembles unknown numbers on the Don't Give-a-Number task, these expressions differ on the standard Give-a-Number methodology. When children 2 years of age and older are asked to give an unknown number, they give a random amount, whereas they generally give all objects in response to requests for *all* (Barner et al., 2009).

To summarize, children at all knower-levels interpreted numbers differently from existential forms like *a* and *some*, whether these numbers were known or unknown. Thus, contrary to some previous proposals (Carey, 2009; Clark, 1970; Clark & Nikitina, 2009), children do not begin with the assumption that number words are interpreted like existential quantifiers. Instead, our data are compatible with the idea that 2- and 3-year-olds treat number words as members of a class of contrasting alternatives, such that *not giving two* leaves open alternative responses like *giving three* or *giving one*. While this result does not speak to whether children initially assume number words denote either exact or unique cardinalities (Sarnecka & Gelman, 2004), it does suggest that children use the structure of the count list to constrain their interpretation of unknown number words, compatible with Quinian bootstrapping. In Experiment 2, we developed a method that allowed us to further explore this idea, to test whether children not only believe that numbers belong to a class of contrasting alternatives, but also that these alternatives are logically structured, with each number referring to a set that contains all smaller amounts, and no larger ones.

3. Experiment 2

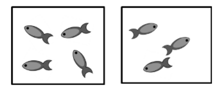
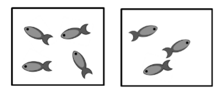
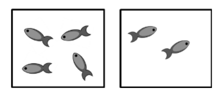
To investigate children's understanding of numerical entailment, we modified Wynn's Point-To-X task, in which children are asked to match a statement – *who has X* – to one of two pictures. To do this, we created a number of different types of affirmative and negative critical trials, described below. Table 1 summarizes these trial types, with the content of the trial depending on each child's highest known number, *N*.

The first experimental trial type tested whether children assume that unknown numbers contrast with known numbers, and therefore sought to replicate Wynn (1992). On these *contrast* trials children were shown two quantities and asked about a number above their knower level, which corresponded to one of the sets. Note that children could

(*footnote continued*)
when all children are included ($\chi^2 = 65.10$, $p < 0.001$), with a significant interaction by Number Knowledge ($\chi^2 = 19.11$, $p < 0.001$). The effect of Existential was separately significant in separate models of CP-and subset-knowers, and marginal for the non-knowers (non-knowers: $\chi^2 = 2.95$, $p = 0.086$; subset-knowers: $\chi^2 = 45.52$, $p < 0.001$; CP-knowers: $\chi^2 = 19.84$, $p < 0.001$).

[8] Within the subset-knowers, the same effect separately held at each knower-level: 1-knowers: $\chi^2 = 9.30$, $p = 0.002$; 2-knowers: $\chi^2 = 19.01$, $p < 0.001$; 3-knowers: $\chi^2 = 12.76$, $p < 0.001$). The two 4-knowers were not analyzed as a separate group.

Table 1
Trial types and subtypes in Experiment 2. For subset-knowers, *N* is the largest number the child knows. For CP-knowers and adults, *N* = 5. With one exception, there were two trials of every type (see note c below). Sample displays show examples of the pictures and corresponding prompt for the first subtype of trial within each type (Control, Contrast, and Split), based on what a 3-knower would see.

Choices	Prompt	Correct[a]	Sample display	Sample prompt
Control trials[b]				
N vs. N + 1	N	smaller		*Can you find the one with* ***three*** *fish?*
N − 1 vs. N	N-1	smaller[c]		
	not(N-1)	larger		
N + 1 vs. N + 2	N + 2	larger		
	N + 1	smaller		
Contrast				
N vs. N + 1	N + 1	larger		*Can you find the one with* ***four*** *fish?*
	not(N + 1)	smaller		
N vs. N + 2	N + 2	larger		
	not(N + 2)	smaller		
Split				
N − 1 vs. N + 1	N	larger		*Can you find the one with* ***three*** *fish?*
	not(N)	smaller		
N vs. N + 2	N + 1	larger		
	not(N + 1)	smaller		
N + 1 vs. N + 3	N + 2	larger		
	not(N + 2)	smaller		

[a] Responses were coded in terms of whether the participant chose the larger quantity or the smaller quantity.
[b] 1-knowers did not receive any control trials with *N* − 1 (which would be *zero*) as a probe.
[c] There were four trials of this stimulus type. There were two trials for each of the other stimulus types.

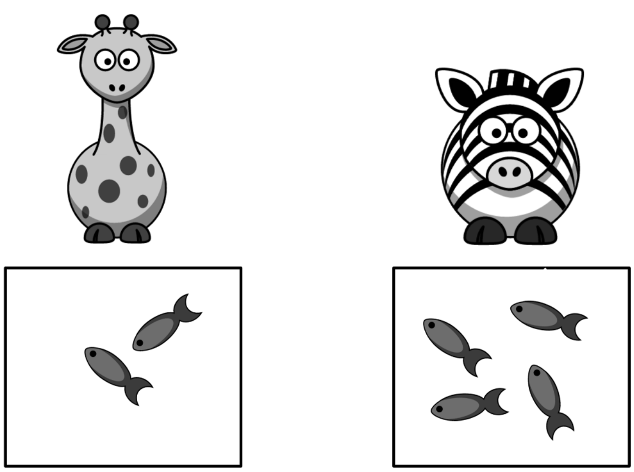


Fig. 2. A representative stimulus array from Exp. 2. The subject was told, "Look! My friend has X fish. Can you find the one with X fish?" (positive prompt) or, "Look! My friend does *not* have X fish. Can you find the one that does *not* have X fish?" (negative prompt).

solve this task using only the principle of contrast, as long as they knew the label of the smaller set. For example, a 2-knower presented with 2 vs. 4 (see Fig. 2), and asked *who has four,* might reason that the speaker intended to indicate the larger set, since otherwise they would have asked *who has two* (Condry & Spelke, 2008; Wynn, 1992). On other trials, we asked children *who does **not** have four*. This question is harder, because it requires the same reasoning to determine that *four* refers to the larger set, and then an additional step to figure out that *not four* refers back to the smaller set of 2. This task may not be easy for 2- and 3-year-olds who have only recently learned the meaning of *not* (Feiman et al., 2017).

To investigate children's understanding of entailment, we introduced *split* trials, in which children also saw two sets, but now were asked about a number in between, corresponding to neither set. For example, a child presented with 2 vs. 4 was asked *who has three* (or *who does not have three*). Since both sets were incompatible with the mentioned number, relying on contrast alone could not determine the correct choice, since both *two* and *four* contrast with *three*. Understanding the asymmetric entailment of number, on the other hand, could guide a child's choice. If I have four quarters, my friend has two, and you need to borrow three quarters for the parking meter, the correct answer to the question, *who has three*, is me and not my friend. Similarly, the correct answer to the question, *who does not have three,* is my friend and not me. More generally, given a larger set and a smaller set and asked who has an unknown number, asymmetric entailment guarantees that the larger set is always the better choice, since the larger set will always contain the smaller one. If asked who *does not* have an unknown number, asymmetric entailment reverses under negation, and similarly guarantees that the smaller set is the better choice. Understanding entailment – and that numbers respect it – would thus predict choosing the larger set on affirmative trials. Understanding how negation reverses entailment would further predict choosing the smaller set on negative trials. If children differentiate between affirmatives and negatives, it would also rule out a simple response bias to always choose either the smaller or the larger set for all prompts.

Critically, to exhibit knowledge of asymmetric entailment, children do not need to know the exact meanings of any of the numbers involved – neither the mentioned number, nor the numbers corresponding to the displayed sets. Nevertheless, this does not guarantee that they'll assume that all numbers exhibit asymmetric entailment: It's possible that children assume that entailment relations hold only between numbers they know, and make no assumptions about unknown numbers. However, it is also possible that they assume that entailment relations hold among all members of the count list, known or unknown. To explore this, we presented children with three different subtypes of *split* trials. These trials varied in how many of the numbers involved were unknown and were above a child's knower level, N. In the first subtype (N − 1 vs. N + 1 trials), the smaller of the two quantities (N − 1) was within their knower level, as was the number word (*N*) they were asked about, while the higher quantity matched an unknown number (N + 1). For example, a 2-knower would see 1 vs. 3 fish, and be asked to find who *has two*. In the second subtype (N vs. N + 2), only the smaller quantity corresponded to a known number. A 2-knower would see 2 vs. 4 fish, and be asked who *has three*. Finally, in the third subtype (N + 1 vs. N + 3), all numbers were beyond children's knower level. For a 2-knower, this would be a trial with 3 vs. 5 fish, and they would be asked who *has four*. We reasoned that if children assume that entailment relations hold among all members of the count list, they should choose the larger set on affirmative trials and the smaller set on negative trials across all cases. However, if they assume that entailment relations only hold among known numbers, they should only succeed when some of the numbers are known.

3.1. Method

3.1.1. Participants

We tested 133 English-speaking children, ages 2;8–5;0 (*M* = 3;8, 69 boys) with the goal of testing approximately 20 participants at each knower-level. The Give-a-Number task identified 72 subset-knowers: 23 1-knowers (2;9–4;4, M = 3;4, 11 boys), 28 2-knowers (2;8–4;4, M = 3;3, 13 boys), 21 3-knowers (2;9–4;5, M = 3;5, 10 boys), 7 4-knowers (2;0–4;3, M = 3;5, 2 boys), and 54 CP-knowers (2;9–5;0, M = 4;2, 33 boys). An additional 18 children were excluded for completing fewer than half the trials (7: two 1-knowers, one 3-knower, and four CP-knowers), experimenter error (2), or inability to assign knower-level (9). Children were tested in the lab or at daycares in San Diego, CA, and Comox, British Columbia. Twenty-one adults (18–23, M = 21, 3 men) participated in exchange for course credit.

3.1.2. Materials and procedure

To begin, we assessed each child's knower-level using Wynn's (1990) Give-a-Number task. All participants then completed a forced-choice task based on Wynn's (1992) Point-To-X task, in which they were asked to identify which of two animals has a specific quantity of fish (see Fig. 2). A full list of trials is presented in Table 1. Participants were randomly assigned to one of two quasi-random trial orders. In both orders, the number of fish depicted in the two images was counter-balanced across trials. Also, half of the trials of each type appeared in the first half of the experiment and half appeared in the second half of the experiment.

There were three trial types (Table 1). *Control* trials validated which numbers were known and unknown. Some tested the child's knowledge of known numbers – e.g., 2-knowers were asked to find the animal who has *two* fish, given a display in which only one animal had exactly two and the other had exactly one. Other control trials, replicating Wynn (1992), tested whether 'unknown' numbers were actually unknown – e.g. 2-knowers were shown an animal with three fish and an animal with four fish, and were asked either who has *three*, or who has *four*. Following Wynn, we expected 2-knowers to choose randomly between the sets.

Contrast trials presented children with pictures of two quantities, and queried them on a number word corresponding to one of the quantities, but above their knower level. There were three subtypes of *contrast* trials. One showed sets of N vs. N + 1 and queried on *N* + 1 or *not N* + 1; another showed sets of N vs. N + 2 and queried on *N* + 2 or *not N* + 2. For example, on a N vs. N + 2 contrast trial, 2-knowers were shown an image like the one in Fig. 2 and were asked to find the animal who *has four fish* (or, alternatively, the animal who *does not have four fish*).

Split trials presented children with two non-consecutive quantities and queried them about an intermediate number word. There were three subtypes of *split* trials. One showed sets of N − 1 vs. N + 1 and queried on *N* or *not N*; another showed sets of N vs. N + 2 and queried on *N* + 1 or *not N* + 1; and finally, one showed two unknown numbers, N + 1 and N + 3, and queried children on *N* + 2 or *not N* + 2. For example, on a N + 1 vs. N + 3 trial, 1-knowers were shown an image like the one in Fig. 2, with two and four fish, and were asked to find the animal who did (or did not) have *three* fish.

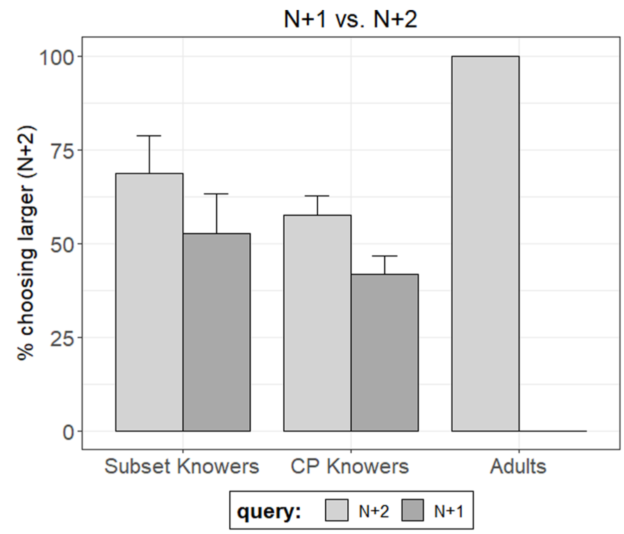


Fig. 3. Participants' responses on *control* trials querying unknown numbers (children are shown N + 1 vs N + 2 and queried on either *N* + 1 or *N* + 2). The y-axis indicates the percent choosing the larger set (i.e. N + 2). For subset-knowers, N is the largest known number. For CP-knowers and adults, N = 5. Error bars show 1 standard error from the mean.

3.2. Results

As in Experiment 1, we grouped different subset-knowers together for the analyses below. Note that in Experiment 2 the content of trials was yoked to each child's knower level, so children at different levels were tested with different stimuli (e.g., different number words and different set sizes), making comparison across levels less interpretable. Nevertheless, for interested readers we again include an exploratory breakdown of performance by knower level in footnotes, with figures in Appendix B. For both subset- and CP-knowers, we proceed with separate analyses of the three trial types and their subtypes.

Control Trials. We first analyzed those control trials in which the queried quantities were known (i.e., trials in which participants saw either N vs. N + 1 and were asked *who has N*, and those where they saw N − 1 vs. N and were asked both *who has* and *who does not have N − 1*). Collapsing across these three trial types, we built separate mixed effects logistic regression models for subset- and CP-knowers with random intercepts for subject and items.[9] These revealed that all subject groups correctly answered these control trials at above-chance rates, confirming that they understood the task (Subset-knowers: M = 81.57%, $Z = 3.28$, $p = 0.001$; CP-knowers: M = 71.91%, $Z = 5.43$, $p < 0.001$; adults: M = 100%). We separately analyzed the control trials that asked children about unknown quantities, presenting them with N + 1 vs. N + 2 and asking either *who has N* + 1 or *who has N* + 2. Fig. 3 shows the results of these unknown control trials. While Wynn (1992) found chance performance on these trials, she collapsed performance on queries asking for N + 1 and those asking for N + 2. We analyzed the two queries separately and found that participants were more likely to choose the animal with N + 2 when asked for the animal who has N + 2 than the animal who has N + 1 (Subset-knowers: $\chi^2 = 13.09$, $p < 0.001$; CP-knowers: $\chi^2 = 5.40$, $p = 0.020$). In particular, while both Subset- and CP-knowers chose randomly when asked *who has N* + 1, Subset- (but not CP-knowers) chose N + 2 when asked *who has*

[9] Omnibus analyses across trial types are both less theoretically relevant and not statistically appropriate. In order to keep the task at a manageable length for two-year-olds, we eliminated some trials from the full crossing that were not informative.

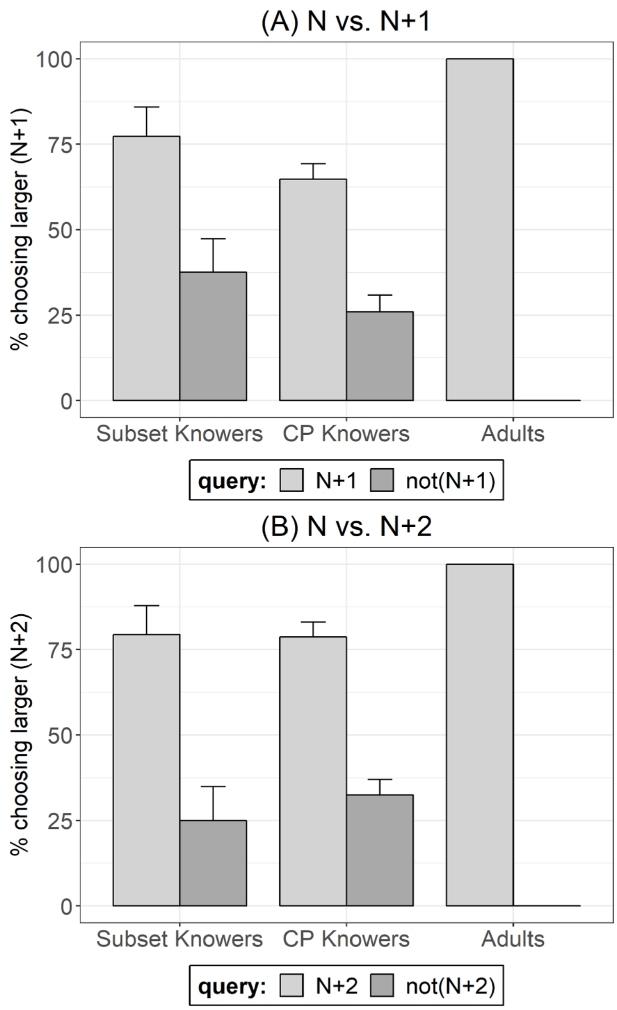


Fig. 4. Percentage of participants choosing the larger option in each of the two types of *contrast* trials: A (N vs. N + 1) and B (N vs. N + 2). For subset-knowers, N is the largest known number. For CP-knowers and adults, N = 5. Error bars show 1 standard error from the mean.

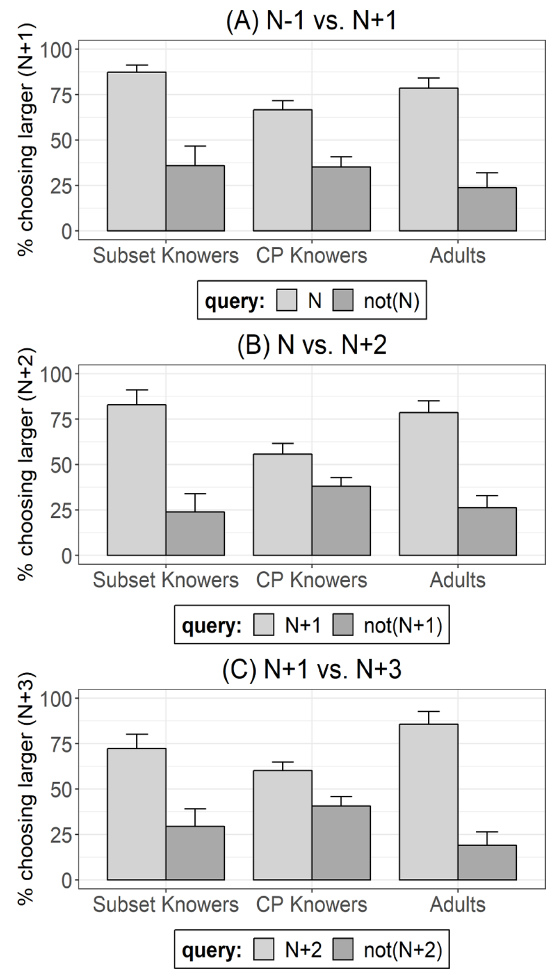


Fig. 5. Percentage of participants choosing the larger option in each of the three types of *split* trials (A–C): A (N − 1 vs. N + 1), B (N vs. N + 2), and C (N + 1 vs N + 3). For Subset-knowers, N is the largest known number. For CP-knowers and adults, N = 5. Error bars show 1 standard error from the mean.

N + 2 (Z = 3.74, p < 0.001). We return to this discrepancy in the discussion. [10]

Contrast Trials. These trials were coded in terms of whether the subject chose the animal with the larger quantity of fish (Fig. 4). This was the correct answer for half of the trials, whereas for the other half, the correct answer was the smaller quantity (Table 1). Of key interest is how children perform on each subtype. Thus, we report results by subtype, using separate logistic mixed effects models throughout. These models included a fixed effect of polarity (negative vs positive) when comparing the two trial types to each other, or an intercept term, when comparing performance to chance. All of the models included random intercepts by subjects and items, and random slopes by the effect of negation, where this was a fixed effect.

The overall pattern of results for each of the individual subtypes of contrast trial was the same. On N vs. N + 1 trials (where *N* was the child's knower-level), each group was more likely to choose the animal with the larger quantity (N + 1 fish) when asked for the animal who *has* N + 1 than when asked for the animal who *does not have* N + 1 (Subset-knowers: χ^2 = 35.09, p < 0.001; CP-knowers: χ^2 = 32.17, p < 0.001). Similarly, on N vs. N + 2 trials, all subject groups were more likely to choose the animal with the larger quantity (N + 2 fish) when asked for the animal *who has N* + 2 than the animal who *does not have N* + 2 (Subset-knowers: χ^2 = 17.19, p < 0.001; CP-knowers: χ^2 = 23.21, p < 0.001).

Split Trials. These trials provided a critical test of whether children were sensitive to the entailment relations between numbers. As with the Contrast trials, we again coded whether subjects chose the animal with the larger quantity of fish (Fig. 5). Recall that the first measure of understanding entailment is systematically choosing larger quantities in response to positive prompts. We analyzed the results using separate regressions for each subtype of Split trial and each group, to check whether children's performance on positive prompts differed from chance. When asked who has a known number, *N*, all groups chose the larger quantity more often than chance (Subset-knowers: Z = 4.24, p < 0.001; CP-knowers: Z = 2.86, p = 0.004; adults: Z = 3.46, p < 0.001). More strikingly, Subset-knowers did the same thing on both of the other subtypes, when asked *who has N* + 1 (Subset-knowers: Z = 4.52, p < 0.001; CP-knowers: Z = 1.07, p = 0.286; adults: Z = 2.48, p = 0.013) and *who has N* + 2 (Subset-knowers: Z = 4.57, p < 0.001; CP-knowers: Z = 2.04, p = 0.042; adults: Z = 3.07,

[10] Note that CP-knowers' performance is generally less robust than subset-knowers'. We see two possible reasons: (1) there are fewer CP-knowers, and (2) they saw larger, harder to discriminate sets and were queried on higher, non-subitizable numbers (e.g. a control trial showing N v N + 1 and querying *N* for a CP-knower translates into a display of 5 vs. 6, querying *five*).

$p = 0.002$), suggesting that their understanding of entailment may extend to numbers in their count list beyond those for which they have acquired stable exact meanings.

Children also understood how negation reversed the entailment pattern – both groups of children were less likely to choose the larger quantity when asked who does not have a number than asked who has it (all *p*s < 0.05), showing that they differentiated the negative and affirmative requests. Since children picked the larger quantities for affirmative prompts, a more stringent test is not merely whether they differentiated negatives from affirmatives, but whether they chose the larger set less than 50% of the time when asked who does not have a number. As in the case of entailment understanding in positive contexts, children's understanding of entailment under negation extended beyond known numbers. Children were less likely than chance to choose the larger quantity when asked for the animal *who does not have N* on N − 1 vs. N + 1 trials (Subset-knowers: $Z = -2.67$, $p = 0.008$; CP-knowers: $Z = -2.31$, $p = 0.021$; adults: $Z = -2.27$, $p = 0.023$), when asked *who does not have N* + 1 on N vs. N + 2 trials (Subset-knowers: $Z = -4.89$, $p < 0.001$; CP-knowers: $Z = -2.25$, $p = 0.025$; adults: $Z = -2.95$, $p = 0.003$), and even when asked *who does not have N* + 2 on N + 1 vs. N + 3 trials, in which all quantities were unknown (Subset-knowers: $Z = -3.99$, $p < 0.001$; CP-knowers: $Z = -1.93$, $p = 0.054$; adults: $Z = -2.11$, $p = 0.034$). For example, 2-knowers who were presented with a set of three and a set of five were more likely to choose the set of five when asked, *Who has four fish*?, but more likely to choose three when asked, *Who does not have four fish*?

3.3. Discussion

Both CP- and subset-knowers demonstrated a clear understanding that *having* some number of items is satisfied by larger quantities than what that number refers to, while *not having* that number is satisfied by smaller ones. In fact, we found that subset-knowers respected these entailment relations even when all quantities involved were unknown, in both positive and negative contexts, a finding of particular significance to explaining how children could learn unknown number words. These findings suggest that knowledge of numerical entailment precedes children's acquisition of the precise cardinal meanings of number words. The fact that children selectively chose larger quantities for *N*, but smaller quantities for *not N,* further suggests that their success in positive contexts cannot be due to simply relying on a default strategy to choose the bigger quantity.

While there are alterative explanations for children's performance on some kinds of trials, none of these extend to all trials. Given that children's performance looks the same across the board, the simplest explanation is that their judgments across trials reflects one common capacity. For example, on N + 1 vs. N + 3 *split* trials – when all numbers are unknown, children might in principle just associate numbers higher on the count list with larger quantities (the "later-greater" principle; see Carey, 2009), choosing the larger set for N + 2 because it is a higher number than they know. However, children do not typically acquire the later-greater principle until well *after* they are CP-knowers (Le Corre, 2014). Moreover, relying on the later-greater principle could not explain children's performance on N − 1 vs. N + 1 trials, when queried on *N*. Later-greater is a principle about the meanings of *unknown* numbers; it does not apply here, because children know the exact meaning of *N* (and therefore that it does not exactly match the set of N + 1), and yet choose N + 1 nevertheless. A different explanation is possible for children's performance on the *split* trials where they saw N vs. N + 2 and were queried on *N* + 1: it is possible to choose the larger set on the basis of contrast rather than entailment. That is, if children know neither *N* + 1 nor *N* + 2, they may work through the same contrastive logic when asked for *N* + 1 as if they had been asked for *N* + 2, thinking that because *N* + 1 does not refer to the set of N (for which they know the corresponding number), it must label the other, larger set. While contrast and entailment coincidentally lead to the same choice in this case, the same is not true in the other two subtypes of *split* trials. When children see N + 1 and N + 3 and are queried on *N* + 2, contrast does not help because they know none of the numbers involved. When children see N − 1 and N + 1 and are queried on *N*, they know the exact meaning of *N* and therefore know that it contrasts with both (the also known) N − 1 set, and the N + 1 set (even if they do not know which word applies to that one). While different alternative strategies might explain performance on some kinds of trials but not others, we found that on all three subtypes of *split* trials, subset-knowers consistently chose the larger set following affirmative prompts and the smaller set following negative prompts. The simplest explanation is that they applied the same understanding of numerical entailment, and of its reversal under negation, across all of these trials.

One surprising feature of these data is children's performance on the N + 1 vs. N + 2 *control* trials. These trials were meant to replicate Wynn (1992), who found that subset-knowers were at chance when presented with N + 1 vs. N + 2 and were prompted to point to either of the corresponding numbers. Wynn grouped both *N* + 1 and *N* + 2 prompts together and found chance performance. In contrast, we looked at each prompt separately, and found that while subset-knowers pointed randomly when asked who *has N* + 1, they tended to choose N + 2 when asked who *has N* + 2. While one somewhat unlikely explanation of this finding is that N-knowers know more about *N* + 2 than *N* + 1, another possibility is that two factors play a role in children's judgments. First, as we found on both *contrast* and *split* trials, children tend to choose larger quantities when asked *who has x* when they do not know exactly what quantity *x* picks out. This factor alone explains children's performance when asked about *N* + 2. Second, some subset-knowers may have begun to map *N* + 1 to the cardinality it denotes, even if this knowledge is not so robust that they consistently give exactly and only that amount when asked to *give N* + 1 (see Barner & Bachrach, 2010; Gunderson, Spaepen, & Levine, 2015). If these children tend to point to N + 1 when asked who *has N* + 1, while other children continue with their default behavior and choose N + 2, the average of the two responses may look like random responding. While this is speculative, it converges with recent evidence that subset-knowers have some partial knowledge of numbers above their knower-level (Barner & Bachrach, 2010; Gunderson et al., 2015; Wagner, Chu, & Barner, 2018), and points to the importance of characterizing the nature of this knowledge.

4. General discussion

In two experiments we found evidence that subset-knowers – i.e., children who have not yet learned the exact meanings of numbers past *three* or *four* – use knowledge of abstract properties of number to restrict inferences about the meanings of specific unknown number words (i.e., number words for which they do not yet have a reliable, exact, meaning). Experiment 1 suggested that subset-knowers use contrast and the structure of the count list to distinguish their interpretation of unknown numbers from that of existential quantifiers like *some* or *a*, contrary to Carey (2009), Clark (1970), and Clark and Nikitina (2009). When asked to *not give some* objects or *an* object, subset knowers preferred to give nothing, whereas when asked to *not give* a number beyond their knower-level, these same children gave some positive amount. Children assumed that a request to *not give N* can be satisfied by giving some other number (e.g., that a request to *not give five* can be satisfied by giving one, two, three, or four), consistent with their knowing that numbers belong to a set of contrasting lexical alternatives (Wynn, 1992). This was true even for non- and 1-knowers, when they were asked not to give higher numbers that they did not know the exact meaning of, like *two* or *five*; they provided some positive amount when asked to not give a number, but gave nothing when asked not to give *a* or *some*. Thus, children used their knowledge that number words contrast with each other to differentiate unknown numbers from existential quantifiers.

Further exploring how knowledge of the number system constrains children's inferences about individual number meanings, Experiment 2 found that subset knowers assume that expressions containing numbers beyond their knower-level fall on an entailment scale, and that this scale reverses under negation. For example, when 2-knowers were presented with a comparison of 1 vs. 3 fish and asked who *has two fish* they preferred the larger set, whereas they preferred the smaller set when asked who *does not have two fish*. In fact, subset knowers exhibited this behavior even when all numbers were outside their Wynn knower-level: They judged that a request for $N + 2$ was satisfied by N + 3, whereas a request for *not* $N + 2$ was satisfied by N + 1. This suggests that, when told that a set contains N, children assumed that whatever N means, it can be satisfied by an amount of N or more, but not by an amount of less than N, and therefore that choosing a larger quantity over a smaller one guarantees a correct response. In sum, these results provide strong evidence that children draw on their knowledge of both the count list and an abstract understanding that larger sets contain smaller sets to make logical inferences when interpreting number words, even before their adult-like exact meanings are fully known.

Before we explore the significance of these findings for the acquisition of number words in particular and abstract concepts more generally, we first address alternative explanations for each of the two experiments.

4.1. Alternative explanations

In Experiment 1, children were asked *not to give* some number *N* of, e.g., bananas. In response, they gave varying amounts of bananas at all knower-levels (see Fig. 1). Our analysis of this behavior is that it not only reflects knowledge of contrast, but also that children know that numerals contrast in a particular way, by denoting alternative cardinal values, such that a request to *not give N* is interpreted as a request to give some other number. However, a possible deflationary interpretation of this result is that giving some positive number of bananas is just children's default behavior in the context of a giving task in which bananas are mentioned. On this account, it is not necessary to invoke knowledge of contrast or alternatives.

Several results speak against this explanation. First, children did not just give bananas any time they were mentioned, as evidenced by the trials on which *some* and *a* were tested. Children at every knower-level gave zero bananas more than half the time when asked either to *not give a* or *not give some bananas*, interpreting these as requests to give none (see Fig. 1). A more qualified deflationary account might argue that children are capable of giving zero in response to requests for negative existentials like *a* and *some*, which they understand, but revert to their default behavior when the request is confusing. However, this account would predict that children might behave differently for numbers within their knower-level limit compared to unknown numbers. Against this, however, we found that children's reluctance to give zero in response to numbers did not differ between numbers above and within their knower level. Children simply differentiated existentials from all numbers, at all knower-levels. They interpreted requests for *not N* to mean that they should give some quantity, rather than nothing, but only when *N* was a number word.

In Experiment 2, children chose pictures with more objects in response to affirmative prompts than they did in response to negative prompts. While we argue that this is because of their understanding of entailment, and of how entailment reverses under negation, a possible deflationary account is that children used some form of heuristic when performing our task, and interpreted negation to mean *less than* or *small amount*. While this idea can describe our data from Experiment 2, it nevertheless faces several problems. First, in most cases negation is not associated with small amounts. For example, a statement like, "That is not a cat", is neutral with respect to quantity, and instead makes reference to a property that is true or false of an object. It does not mean "less than a cat" or "a small amount of cat", but instead that the referent in question is something other than a cat. For adults, negation generally only identifies smaller numbers when used in combination with number words, precisely because of how it reverses entailment along the number scale. These facts mean that, if children do rely on a heuristic that falls short of full-fledged entailment, such a heuristic would need to have almost exactly the same content as entailment and be derived from exactly the same evidence – i.e., from how adults use number words, how they use negation, and how these two compose semantically.

Further, data from Experiment 1 provide empirical evidence against this possibility, since children did not exhibit a preference to give especially small numbers in response to negative requests. When told not to give N objects, children did not asymmetrically give numbers greater or smaller than N (see Appendix A). For example, in response to a request to *give everything, but not two bananas* children were as likely to give one banana as to give all five. It is interesting that entailment relations did not seem to play a role in children's behavior in Experiment 1. One likely reason for this finding is that sentences in Experiment 1 were imperatives (i.e., requests), rather than statements about the world. Although entailment relations hold between descriptions of states of the world, previous studies have reported that imperatives fail to generate scalar implicatures, a result which is also consistent with a relative insensitivity to asymmetric entailment relations (see Singh, Wexler, Astle-Rahim, Kamawar, & Fox, 2016, for discussion). At the very least, Experiment 2 shows that children do not compute only simple exclusion inferences when interpreting negated numbers – i.e., they do not rely simply on contrast or mutual exclusivity, which each permit symmetrical negation of both higher and lower alternatives. Instead, their inferences reflect asymmetric negation of alternatives, with the direction of the asymmetry changing under negation, consistent with knowledge of entailment.

4.2. Negation and entailment in childhood

The finding that children understand how negation reverses the entailment relations of numbers in their count list supports two key conclusions. First, children understand how negation affects entailment at most only six months after they understand negation words as logical at all. Second, children's understanding that larger sets contains smaller sets, and how this constrains the meanings of individual number words, provides a robust conceptual scaffold that children could use to learn the meanings of number words. We address each of these ideas in turn.

While previous studies have found that children as young as 2 years of age comprehend the logical meaning of words like *no, not,* and *didn't* (Austin et al., 2014; Feiman et al., 2017; Reuter et al., 2018), these studies have focused on children's comprehension of negation in the service of exclusion inferences between one of two options. For example, given two options (e.g., a bucket and a truck), and told that a hidden ball is *not* in one of them, 2-year-olds search in the other. But being able to exclude a possibility does not necessarily mean being able to exclude its implicit logical consequences. Our study provides the first evidence that young children reason from an utterance through to its unstated entailments, a deductively valid and therefore especially powerful example of how children can gain new knowledge without new experience, or "learning by thinking" (Lombrozo, 2016; Walker, 2015).

While our work shows that very young children readily interpret negation in the context of a scale, several open questions remain: Do children integrate negation and entailment relations just as readily in scales other than number? Is this integration learned separately and perhaps piecemeal for different scales, or simply comes for free by virtue of both comprehending negation and representing a scale? Do they understand other triggers to reversing entailment, such as conditional statements? For example, do they understand that the statement, *if you have two cookies, you should share them* does not entail that you should share if you only have one cookie? Do children further understand multiple embedded reversals of entailment within a sentence, the way that adults intuitively understand that, ***no** aardvark **without** a keen sense of smell can find food* entails that, ***no** aardvark **without** a sense of smell can find food* (see Icard & Moss, 2014)? What we know from the present study is that children don't require a complete mastery of the semantics of the scale – i.e., the exact meaning of each member – to understand either the entailment relations between members, or how these relations reverse under negation. The tasks that we have developed here provide a method for exploring this issue in further studies, by testing children at the earliest moments they begin to comprehend negation.

4.3. Number word learning as Quinian bootstrapping

The results of this study suggest that children's early hypotheses about the meanings of number words – both known and unknown – may be informed by their membership in the count list and by knowledge of numerical entailment. These results are important because they suggest that these hypotheses may be informed by their understanding of the logical relations between numbers, contrary to most previous accounts. Most recent studies of number word learning have focused on counting and concluded that subset knowers not only do not understand counting (Condry & Spelke, 2008; Le Corre & Carey, 2007; Wynn, 1990, 1992), but also that they do not even understand that counting is relevant to determining cardinality in many contexts. For example, although even some non-knowers can recite a partial count list, subset-knowers do not even *try* to count when asked to give precise numbers (Le Corre & Carey, 2007). Other studies have gone further, to argue that the logic of counting is probably learned some months (or maybe years) after children can reliably use the counting procedure to give sets, suggesting that the logic of counting may be derived following years of using a relatively blind procedure (Davidson et al., 2012; Wagner, Kimura, Cheung, & Barner, 2015; Cheung et al., 2017).

While it may take years for children to acquire the full logic of counting, our study suggests that they may begin learning it very early on, such that this knowledge could inform the acquisition process, in keeping with Quinian bootstrapping. How might knowledge of entailment facilitate number word learning? In a previous study, Barner and Bachrach (2010) proposed that early number words may receive exact interpretations via a form of logical inference that depends on knowledge of entailment – i.e., scalar implicature. Specifically, they proposed that children may initially assign the number *one* a non-exact meaning equivalent to *a*, such that both *a dog* and *one dog* denote singleton dog sets. At this point, though both *a* and *one* denote precise quantities – i.e., singleton sets – their meanings are not strengthened via contrast to other, stronger, expressions. Thus, for example, children initially accept both *a banana* and *one banana* for sets larger than one (Barner et al., 2009). However, once children learn the core meaning of *two* (i.e., doubleton sets), *one* is strengthened by appeal to *two* to mean A SINGLETON, BUT NOT A DOUBLETON, OR MORE.[11] At this point, children are classified by tasks like Give-a-Number as 1-knowers, since they treat *one* as exact, and now deny that there is one banana in a container if there are two or more, despite continuing to agree that there is *a banana* (Barner et al., 2009). More generally, on this hypothesis, children's first number word meanings are equivalent to singular, dual, and trial (see also Almoammer et al., 2013; Marušič, Žaucer, Plesničar, Sullivan, & Barner, 2016), and are strengthened to mean EXACTLY ONE, EXACTLY TWO and EXACTLY THREE via contrast with stronger scale mates – i.e., by a form of scalar implicature. Critically, this type of inference relies on an appeal to entailment: An expression like, *There are two bananas* is strengthened to, *There are exactly two bananas* by ruling out other statements that a speaker might have uttered (e.g., *There are three bananas*), so long as they are not weaker than the original statement (i.e., are not logically entailed by them). Since, *There are three bananas* logically entails *There are two bananas*, the former utterance is negated as a possible meaning when interpreting the latter. More informally, the child reasons that if the speaker had believed that there really are three bananas, then they should have said so, since that claim is strictly stronger than the alternative containing two, despite both being literally true.

Our study provides evidence that is compatible with this conclusion. In Experiment 2, we find that children are able to make entailment inferences for number words before they acquire their exact meanings (before they succeed with N on the Give-N task), a capacity which could be used to derive exactness. By reasoning that a set described as *N bananas* contains at most N bananas (since otherwise the speaker would have used its successor or a higher number), and because N bananas must contain at least N, children could derive an exact meaning – that N bananas is at least N, and at most N bananas. Running the mirror image of this logic for the meaning of *not N bananas* might further cement that *N* means exactly N.

While we have argued that children are able to understand the entailments of expressions containing number words, it is an open question whether they also understand entailment in other domains, and whether this might help them acquire concepts in those domains. Scales abound in our conceptual repertoire and in how it is expressed in language (Horn, 1972). For example, *hot* and *warm* are scale-mates, as are *some* and *all*. Does learning that a pair of words falls on a scale, together with the direction of that scale's entailment, help children figure out the meaning of one word if they know the meaning of the other? The "Point to Not-X" task we developed in Experiment 2 may provide a tool to test children's understanding of entailment on other scales, as well. Coupled with separate tests of whether children have knowledge of each individual concept represented by a scale, this framework may allow further tests of whether understanding entailment precedes the acquisition of individual concepts in domains other than number.

5. Conclusion

This study suggests that when children acquire number words, they use the structure of the count list to differentiate these words from other quantity expressions like *a* and *some*. They also use the mapping between number words and the logical structure of cardinalities to make inferences regarding the entailments of number words, even before they

[11] As noted by Barner and Bachrach, Wynn's Give-a-Number task is conservative and presupposes an exact semantics for number words. According to Wynn and subsequent studies that have used her task, a child is an N-knower if they give N two out of three times when asked for N, and if on two out of three times that they give N it is in response to requests for N. Thus, a child can use N + 1 accurately 65% of the time – or even 100% of the time – might still be classified as an N-knower if for N + 2 they responded randomly and sometimes gave N + 1.

learn their exact meanings. By learning that an expression like *having two* is compatible with having three but not with having one, and that *not having two* is compatible with having one but not with having three, children may also acquire the knowledge necessary for inferring that *two* denotes more than one, but less than three – i.e., exactly two. On this account, children may acquire number word meanings using a form of Quinian bootstrapping, using the abstract relations that hold between concepts to constrain and enrich hypotheses about the concepts themselves. The present study provides the first evidence that children understand a fundamental abstract relation that underlies all of the natural numbers before they know the meanings of these numbers or the logic of counting, and thus that children may bootstrap exact lexical meanings from knowledge of the structures in which they are used.

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Appendix A

See Figs. A1 and A2.

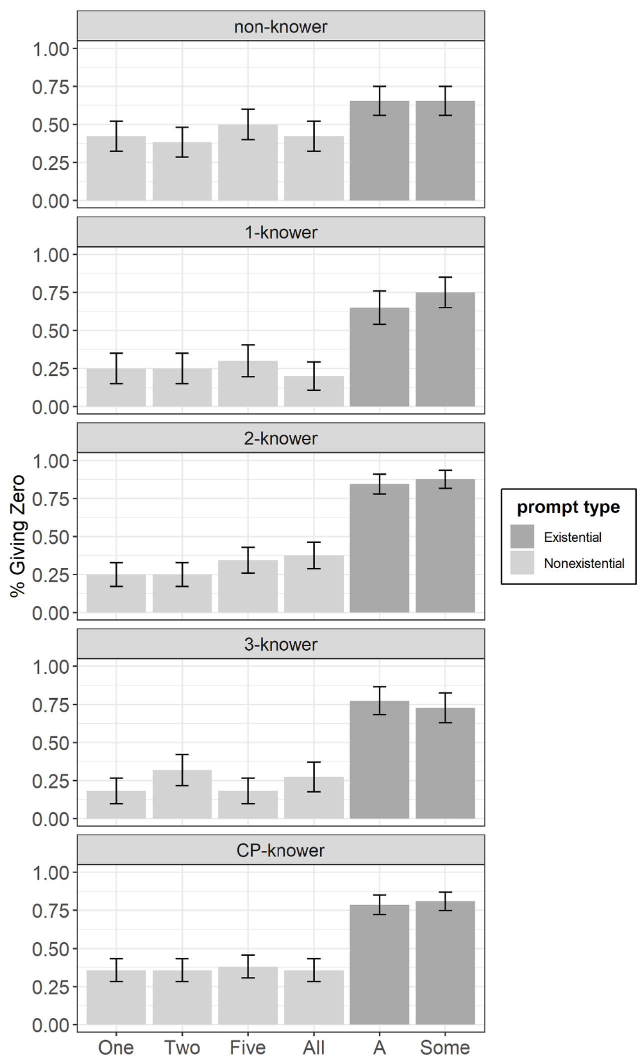

Fig. A1. Proportion of trials on which participants gave 0 of the target items in response to each type of request, broken down by knower-level. Error bars show ± 1 standard error, averaged by participant. The two 4-knowers in the sample are excluded from this breakdown.

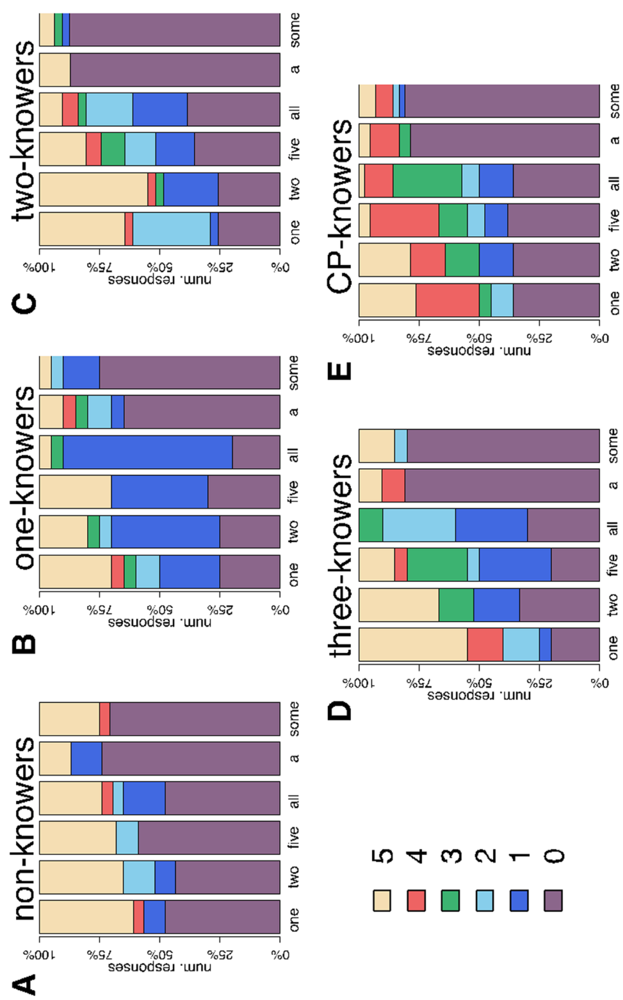

Fig. A2. Proportion of trials on which participants gave 0, 1, 2, 3, 4, or 5 of the target items, for (A) 13 non-knowers, (B) 10 1-knowers, (C) 16 2-knowers, (D) 11 3-knowers, and (E) 21 CP-knowers.

Appendix B

See Figs. B1 and B2.

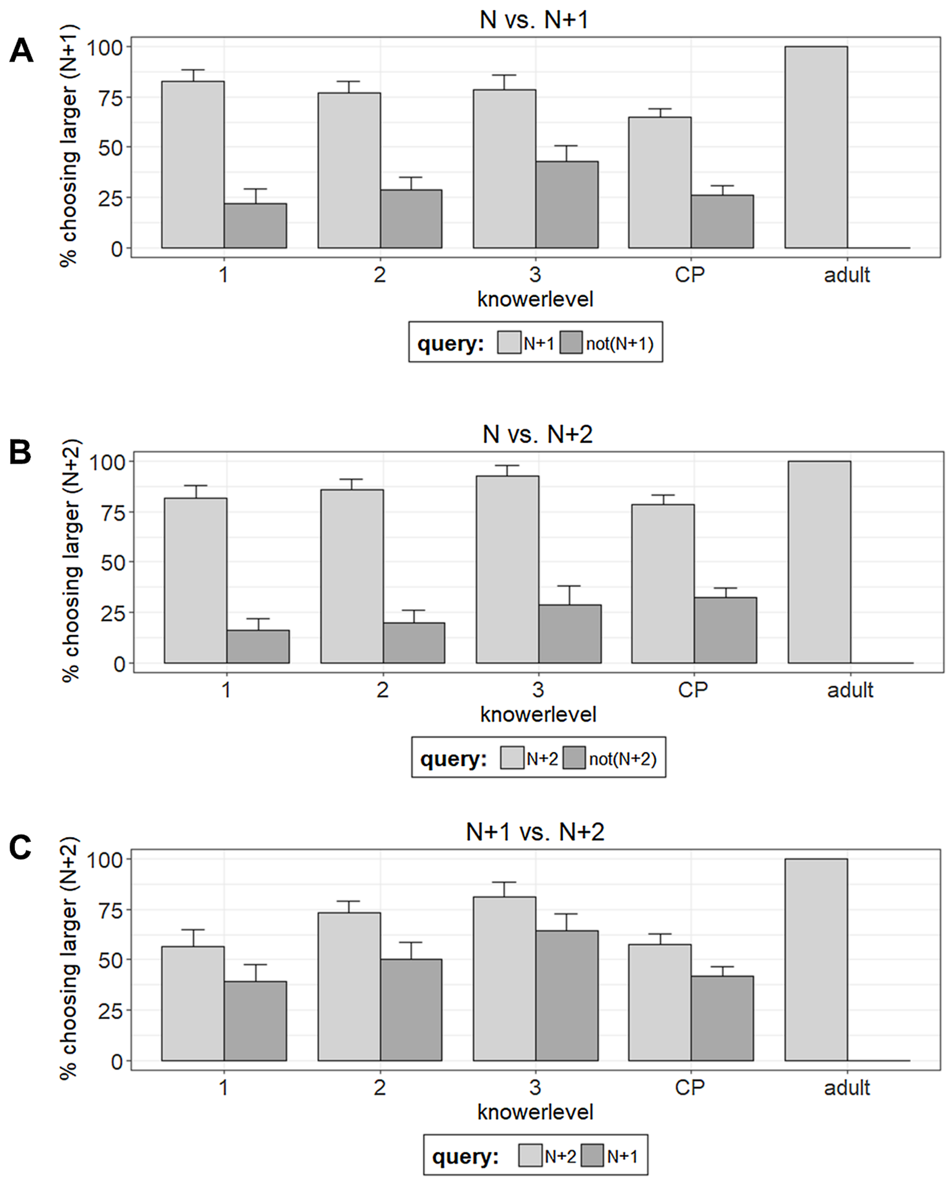

Fig. B1. Percentage of participants choosing the larger option in each of the subtypes of *contrast* trials (A: N vs. N + 1, B: N vs. N + 2) and (C) the unknown number *control* trials in Experiment 2, broken down by knower level. For subset-knowers, N is the largest known number. For CP-knowers and adults, N = 5.

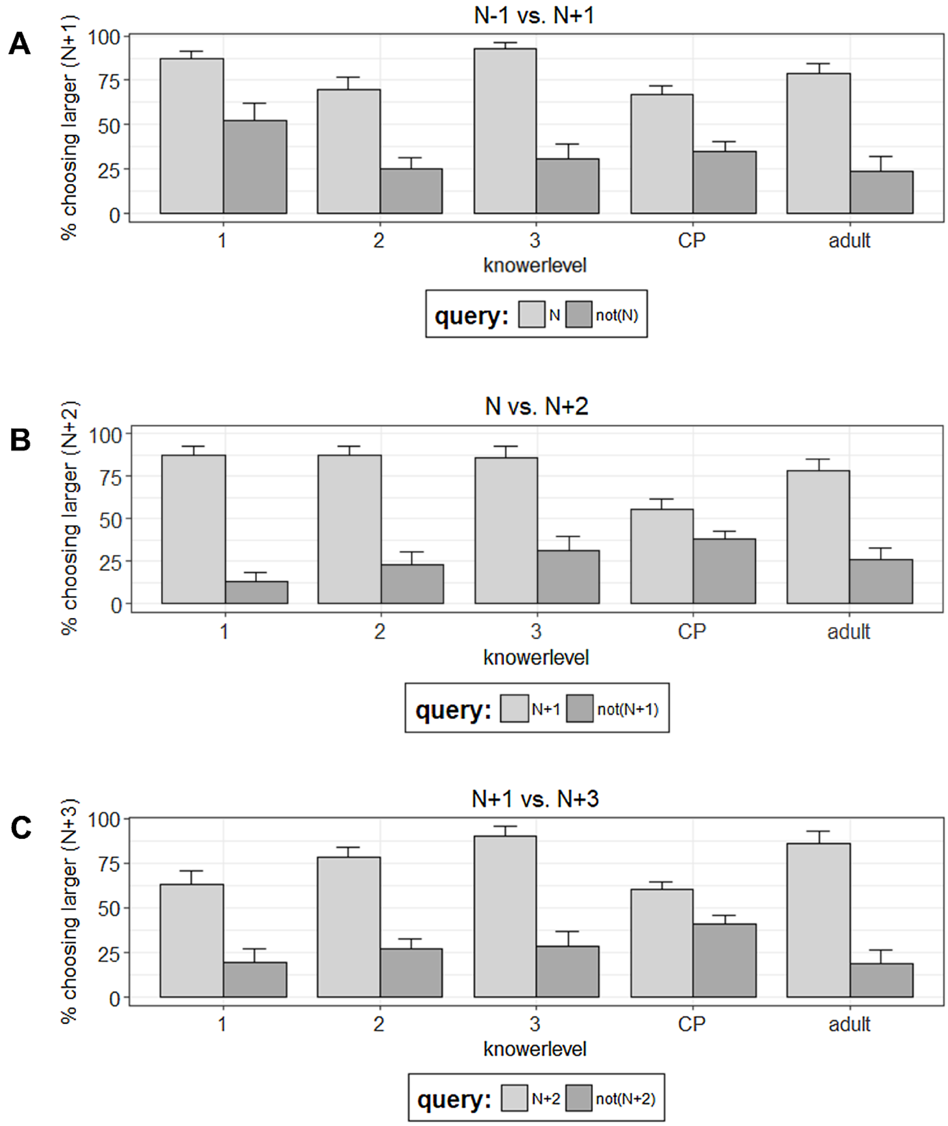

Fig. B2. Percentage of participants choosing the larger option in each of the three types of *split* trials in Experiment 2, broken down by knower level: A (N − 1 vs. N + 1), B (N vs. N + 2), and C (N + 1 vs N + 3). For subset-knowers, N is the largest known number. For CP-knowers and adults, N = 5.

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